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A new constructing approach for a weighted topology of wireless sensor networks based on local-world theory for the Internet of Things (IOT)

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ABSTRACT

In this paper, we propose a new constructing approach for a weighted topology of wireless sensor networks (WSNs) based on local-world theory for the Internet of Things (IOT). Based on local-world theory, an uneven clustering weighted evolving model of WSNs is designed. The definitions of edge weight and vertex strength take sensor energy, transmission distance, and flow into consideration. The vertex strengths drive the growth of topology; meanwhile, the edge weights change correspondingly. Experimental data demonstrate that the WSN topology we obtain has the property of weighted networks of the IOT: the edge weight, vertex degree, and strength follow a power-law distribution. Related IOT research work shows that weighted WSNs not only share the robustness and fault tolerance of weight-free networks, but also reduce the probability that successive node breakdowns occur; furthermore, they enhance the synchronization of WSNs.

1. Introduction

It is well known that a wireless sensor network (WSN) is a self-organization wireless network system constituted by several energy-limited micro sensors under the banner of the Internet of Things (IOT). Nowadays, WSNs are widely used as an effective medium to integrate the physical world and the information world of the IOT [1,2]. In [3], we propose a kind of new web-based method of seamless migration under the IOT, in which the WSN works as a context-aware device. In order to transmit context-aware information (identity, location, etc.) in time and without fault, a good WSN topology should be constructed to ensure efficient data transmission. Afterwards, the routing protocol and application layer could be designed.

Many algorithms have been used in topology control and routing design to balance the energy consumption and prolong the lifetime of WSNs [4–7], such as clustering [8–14], graph theory [15,16], and intelligence computing [17]. At the same time, interdisciplinary achievements are made and then applied to WSNs. In this paper, we propose a new method of constructing a topology based on local-world weighted networks for WSNs of the IOT, which involves a topology evolving model for the WSN, using the theory of weighted complex networks which is widely studied in the field of statistical physics.

Nowadays, though many achievements in the field of complex networks are emerging, only the unweighted small-world theory is widely applied to constructing and optimizing WSN topology. Learning from interdisciplinary researches in recent years, a new approach to construct a WSN topology of the IOT will be presented by us. The definitions of edge weight and vertex strength take the sensor energy, transmission distance, and flow into consideration. The vertex strengths drive the growth of topology; meanwhile, the edge weights change correspondingly. Experimental data demonstrate that the WSN topology we obtain has the property of weighted networks of the IOT: the edge weight, vertex degree, and strength follow a

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power-law distribution. Our related IOT research work shows that weighted WSNs not only share the robustness and fault tolerance of weight-free networks, but also reduce the probability that successive node breakdowns occur; furthermore, they enhance the synchronization of WSNs.

2. Background

Most of the real networks of the IOT, independent of their age, function, and scope, converge to similar architectures [18]; therefore researchers have tried to build a unified model for complex networks in recent decades. In [19], Erdös and Rényi propose a random graph model based on classic graph theory and statistical physics; in [20], the small-world property of complex network is found by Watts and Strogatz (WS), who establish a small-world network model; in [21], Barabási and Albert (BA) build a model which reveals the scale-free characteristic of complex networks; in [22], a weighted network model is created by Barrat, Barthélemy and Vespignani (BBV); this model not only defines the strength of connections, but also takes the change of connection strength into consideration, which makes the model closer to a real network of the IOT.

Nowadays, the BBV model is widely used to analyze real complex networks such as the scientist collaboration network (SCN) and world-wide airport network (WAN) [23]. Similar to the SCN and the WAN, there are numerous nodes and community structures (clusters) in WSNs; important nodes (cluster heads) have more connections than common nodes. Much research on the "energy hole" shows that the data flow on each connection varies considerably in a WSN because of different distances to the sink node [12]. Thus it is not suitable to represent a connection as connected ("1") or connectionless ("0"). Furthermore, global information is limited in WSNs of the IOT: sensors exchange information in their "local world". Overall, weighted networks and local-world theory are appropriate to model WSNs of the IOT.

In [24], Ruela et al. use a genetic algorithm to construct a WSN topology which shows a high clustering coefficient and a short average shortest path; therefore, the energy consumption and delay of the WSN is reduced. But complex network theory plays the role of a network performance analysis tool, rather than an evolving method in this study. In [25], Chen et al. propose a network topology evolving mechanism between clusters based on a random walk. An energy-driven preferential attachment is available. The topology generated by this method has the characteristics of a scale-free network, and the fault tolerance is better. But the topology of the WSN is unweighted, and is formed by cluster heads rather than all of the sensors; furthermore, DEEG [26] fails to solve the energy-hole problem.

3. Basic complex network theory

As we know, real networks are mostly complex systems which contain lots of members and connections. These members are abstracted to nodes and connections are abstracted to edges. If a complex network is weighted, the weight not only represents the existence of a link between nodes, but also describes the property and intensity of the connection. In the SCN, the weights represent the frequency of cooperation between scientists, and in the WAN, the weights represent the number of available seats in flights between two airports.

A basic complex network of this theory can be expressed as a graph $G = (V, E)$; the nodes are denoted as v_i, and the node set is $V = (v_1, v_2, \dots, v_N)$, where $N = |V|$ represents the total number of nodes. The edge from i to j is defined as $e_{ij} = (v_i, v_j),\ i, j \in (1, 2, \dots, N)$. The set of edges is $E = (e_1, e_2, \dots, e_M)$, and $M = |E|$ means the total number of edges. If the network is not oriented, then $e_{ij} = e_{ji}$. $G = (V, W)$ represents a weighted network, V is the set of nodes, W is the set of edge weights, w_{ij} is the weight between i and j, $\langle w_{ij} \rangle$ is the average edge weight of the whole network, and the definitions of other average statistics are the same as those above. The degree of i is defined as the number of nodes connected to it, denoted as k_i, and the vertex strength s_i is defined as the sum of edge weights connected to it:

$$s_i = \sum_{j \in N(i)} w_{ij}, \tag{1}$$

where $N(i)$ is the set of neighbor nodes (directly connected to i). The distribution function $P(k)$ of node degree represents the probability that $k_i = k$ when node i is randomly selected. The probability distribution definitions of other physical statistics are similar to that of $P(k)$, such as the strength distribution $P(s)$ and weight distribution $P(w)$. In the BA scale-free network model,

$$P(k) \sim 2m^2 k^{-\gamma}, \tag{2}$$

where the power-law index $\gamma = 3$, and m is a constant. Many studies show that the degrees of most real networks obey a power-law distribution, and the range of γ is [2, 3].

4. BBV weighted network model and local-world theory

Based on our research work, we know that the evolving topology of the BBV model can be divided into four stages, as follows.

(1) Initialization. The initial network contains N_0 nodes and a few edges ($w = w_0$).

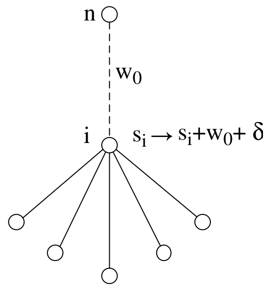

Fig. 1. Change of node strength.

(2) Growth of topology. At each time step, a new node n with m edges ($w = w_0$) joins the existing network.
(3) Preferential attachment. The existing nodes are preferentially attached by m edges in step (2) with probability $\prod_{n\to i}$:

$$\prod_{n\to i} = \frac{s_i}{\sum_j s_j}. \tag{3}$$

(4) Update of strength and weights (Fig. 1).

The addition of edge (n, i) not only changes the strength of i, but also changes the weights between i and its neighbors:

$$w_{ij} \to w_{ij} + \Delta w_{ij}, \tag{4}$$

where

$$\Delta w_{ij} = \delta \frac{w_{ij}}{s_i}. \tag{5}$$

After the update, repeat steps (2)–(4), until the evolution is complete. Let $w_0 = 1$. When $t \to \infty$, the distribution of edge weight is

$$P(w) \sim w^{-\alpha}. \tag{6}$$

The distribution of node degree is

$$P(k) \sim k^{-\gamma_k}. \tag{7}$$

The distribution of node strength is

$$P(s) \sim s^{-\gamma_s}, \tag{8}$$

where

$$\alpha = 2 + 1/\delta, \qquad \gamma_k = \gamma_s = \gamma = (4\delta + 3)/(2\delta + 1). \tag{9}$$

In [27], a local-world evolving network model is proposed by Li and Chen. The study shows that, in a real network, a node can only connect to a special group of nodes rather than any node in the whole network. M nodes are randomly selected from existing nodes as the local world of the new node n, and the preferential attachment probability is defined as

$$\prod_{\text{Local}}(n \to i) = \prod{}'(i \in \text{Local-world}) \frac{k_i}{\sum_{j\,\text{Local}} k_j}, \tag{10}$$

where

$$\prod{}'(i \in \text{Local-world}) = M/(N_0 + t). \tag{11}$$

In the BBV model, m existing nodes from the entire network are selected to connect to the new node n, which is not feasible in WSNs of the IOT due to the limited communication range and the energy of the sensors. So local-world theory is needed; that is to say, n can only connect to the sensors within a specific range. Similarly, in the SCN, scientists tend to cooperate with others who work in the same country or discipline, and in the WAN, the length of a flight is always shorter than the maximum range of a plane, which can be seen as the examples of local world.

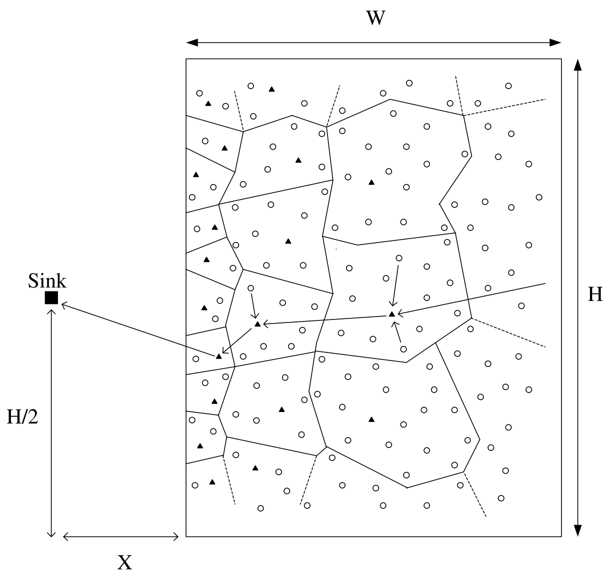

Fig. 2. Distribution map of sink and sensor nodes.

5. New clustering approach based on LW theory and a weighted evolving model

5.1. Network model

As shown in Fig. 2, the sensor nodes are randomly distributed in a $W \times H$ rectangular sensing field. Data are sent to the regional central node (cluster head), and are then transferred to the sink node (Sink). The descriptions and definitions are as follows.

(1) All sensor nodes are isomorphic; they have limited capabilities to compute, communicate,and store data. The set of sensor nodes is defined as $V = (v_1, v_2, \ldots, v_N)$; N is the total number of nodes. i is the unique identifier for a node: $i = 1, 2, \ldots, N$.
(2) The energy of sensor nodes is limited: the initial energy is E_0. Nodes die after exhausting energy entirely. But the energy of the sink node can be added.
(3) The locations of the nodes and the Sink do not change after being fixed. A node cannot obtain its absolute position from its own location device.
(4) A node can vary its transmission power according to the distance to its receiver. The sink node can broadcast message to all sensor nodes in the sensing field.
(5) The distance between the signal source and the receiver can be computed based on the received signal strength.
(6) Regional central nodes are not selected at the beginning; on the contrary, they spring up during the topology evolution. Important nodes have more connections, whose degree and intensity are significantly higher those of than neighbor nodes.

The energy model is the free space model [8]. The energy spent in sending an l-bit packet over distance d is

$$E_{Tx}(l,d) = E_{Tx-\mathrm{elec}}(l) + E_{Tx-\mathrm{amp}}(l,d) = \begin{cases} lE_{\mathrm{elec}} + l\varepsilon_{fs}d^2, & d < d_0 \\ lE_{\mathrm{elec}} + l\varepsilon_{mp}d^4, & d \geq d_0, \end{cases} \tag{12}$$

where

$$d_0 = \sqrt{\frac{\varepsilon_{fs}^2}{\varepsilon_{mp}}}. \tag{13}$$

The energy spent in receiving data is defined as

$$E_{Rx}(l) = E_{Rx-\mathrm{elec}}(l) = lE_{\mathrm{elec}}, \tag{14}$$

where E_{elec} is a fixed energy value spent for sending 1-bit data, and ε_{fs} is the energy coefficient. When the data transmission distance is larger than the threshold d_0, the energy consumption would rise sharply, so the maximum communication radius of common sensor nodes is set to d_0.

Definition 1. The distance between i and Sink is $d(i, \text{Sink})$:

$$d(i, \text{Sink}) \in [X, \sqrt{(H/2)^2 + (X+W)^2}]. \tag{15}$$

The communication radius can be controlled in order to construct a topology with uneven clusters; when i is the cluster head, the optimal cluster radius is $R_{\text{opt}}(i)$:

$$R_{\text{opt}}(i) \sim f_1[d(i, \text{Sink})], \tag{16}$$

where $f_1[d(i, \text{Sink})]$ is an increasing function of $d(i, \text{Sink})$, and

$$f_1[d(i, \text{Sink})] \in (0, d_0). \tag{17}$$

Definition 2. At time t, the weight between i and j is w_{ij}:

$$w_{ij}(t) = \frac{\zeta[E_i(t)E_j(t)]^{\psi}}{\{[d(i,j)]^2\}^{\eta}[T_{ij}(t)]^{\xi}}, \tag{18}$$

where ζ, ψ, η, ξ are non-negative constants, $E_i(t)$ and $E_j(t)$ are residual energy, $d(i, j)$ is the distance between two nodes, and $T_{ij}(t)$ is the data flow [28,29] of the edge (communication link) e_{ij}. Set the distance from i to Sink to be more than that from j to Sink, then

$$T_{ij}(t) \sim f_2\,[d(i, \text{Sink}), t] = \frac{t}{[d(i, \text{Sink})]^2}, \tag{19}$$

where $f_2\,[d(i, \text{Sink}), t]$ is a decreasing function of $d(i, \text{Sink})$ and an increasing function of t. The amount of data is smaller when the edge-end node is farther away from the sink node. As time goes on, the amount of data becomes larger with the increase in the number of nodes. In this definition, the edge weight w_{ij} represents the communication capacity. In Eq. (18), when $d(i, j)$ is long, the data transmission tends to choose a short-distance link. Similarly, when $T_{ij}(t)$ is large, and the communication link is busy, the data transmission chooses a low-load link first. Energy plays a key role in edge weight: when the residual energy of i and j is sufficient, e_{ij} is stronger for data transmission.

Definition 3. The strength s_i of sensor node i

$$s_i = \sum_{j \in N(i)} w_{ij}, \tag{20}$$

where $N(i)$ is the set of neighbor nodes, and k_i is the degree. Like a central city in the WAN and a famous scientist in the SCN, a high-strength sensor can attract more connections. According to the theory of self-learning, airport capacity can be expanded and scientists can improve their level. But the strength of the sensors cannot increase continuously because of the limited energy.

5.2. Evolving mechanism

(1) Network initialization. After broadcasting, the distance to the sink node and R_{opt} can be recorded by every sensor node. Randomly select a subnetwork from the whole network as the initial network: there are $N_0(N_0 < N)$ sensor nodes and e_0 edges $(d_{ij} < d_0)$ in the initial network. Then assign initial values to these edges according to Eq. (18). Reconnection between two nodes is not allowed.

(2) Selection of the local world. At each time step, randomly select $M(M \le N_0)$ nodes from the existing network as the local world of the new sensor node together with m new edges.

(3) Growth of topology. The m edges in step (2) will connect to m nodes. The connection probability is

$$\prod(n \to i) = \prod{}'(i \in \text{Local-world}) \prod{}''[i \in N_{R_{\text{opt}}}(n)] \frac{s_i}{\sum\limits_{j \in [\text{local-world} \cap N_{R_{\text{opt}}}(n)]} s_j}, \tag{21}$$

where $\prod'(i \in \text{local-world})$ is the probability of i being in the local world. $\prod''[i \in N_{R_{\text{opt}}}(n)]$ is the probability of i being within a radius of $R_{\text{opt}}(n)$ of n. $j \in [\text{local-world} \cap N_{R_{\text{opt}}}(n)]$ represents that i meets both of the two above-mentioned conditions. The deployment density of sensor nodes should ensure that each of the m edges can be connected. In step (3), the complexity of the algorithm is $O(N)$.

(4) Dynamic change of weight. Assign values to the m edges according to Eq. (18). For δ in the BBV model, according to the actual situation of the WSN, we have the following.

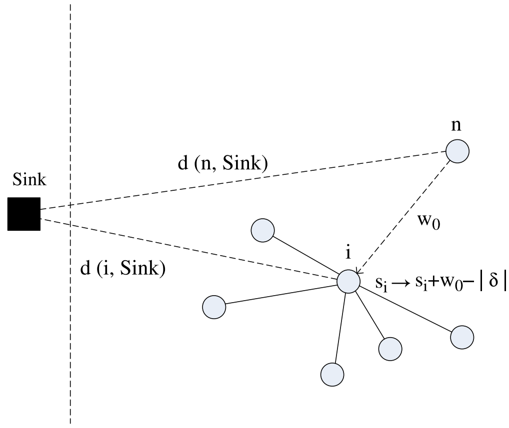

Fig. 3. Change of S_i when the distance from n to the sink node (Sink) is longer than i.

(i) When $d(n, \text{Sink}) > d(i, \text{Sink})$, δ is a negative increment, and

$$|\delta| \sim f_3[w_{ni}(t)], \tag{22}$$

where $f_3[w_{ni}(t)]$ is an increasing function of $w_{ni}(t)$.

The edge weights between neighbor nodes and i change as follows:

$$w_{ij} \to w_{ij} + \Delta w_{ij}, \tag{23}$$

where

$$\Delta w_{ij} = \begin{cases} -|\delta| \dfrac{w_{ij}}{\sum\limits_{j \in N(i), d(j,\text{Sink}) < d(i,\text{Sink})} w_{ij}}, & d(j, \text{Sink}) < d(i, \text{Sink}) \\ 0, & d(j, \text{Sink}) \geq d(i, \text{Sink}). \end{cases} \tag{24}$$

The strength of i changes as follows:

$$s_i \to s_i + w_0 - |\delta|. \tag{25}$$

As shown in Fig. 3, when n is farther away from the sink node than i, n may choose i as the next hop node, which is a new data flow burden for i. The nearer from sink node n is, the heavier the burden for i will be. Similarly, when j is closer to the sink node than i, it shares the burden of data flow for i: the higher w_{ij} is, the more flow e_{ij} shares.

(ii) When $d(n, \text{Sink}) \leq d(i, \text{Sink})$, δ is a positive increment, and similarly

$$\Delta w_{ij} = \begin{cases} |\delta| \dfrac{w_{ij}}{\sum\limits_{j \in N(i), d(j,\text{Sink}) > d(i,\text{Sink})} w_{ij}}, & d(j, \text{Sink}) > d(i, \text{Sink}) \\ 0, & d(j, \text{Sink}) \leq d(i, \text{Sink}). \end{cases} \tag{26}$$

The strength of i changes as follows:

$$s_i \to s_i + w_0 + |\delta|. \tag{27}$$

As shown in Fig. 4, when n is closer to the sink node than i, it shares the burden of data flow for i. When j is farther away from the sink node than i, i shares the burden of data flow for j: the higher w_{ij} is, the more flow e_{in} shares.

Repeat steps (2)–(4) until all nodes in the sensing field join the network topology. In step (4), the complexity of algorithm is $O(1)$, so the overall complexity of algorithm is $O(N)$. When the total number N of sensors is limited, the computation is acceptable for each sensor node. Furthermore, the update of the node is distributed; that is to say, a newcomer joins the network based on local information rather than being controlled by the sink node. On the other hand, the edge weights change with time; the information of edges is so heavy for each sensor that the sink node records them, so it is centralized.

5.3. Solution of distributions

We use the mean-field approach to solve the distributions [30–32]. Given $m < M < N_0 + t$, $N_{l \in [\text{local-world} \cap N_{R_{\text{opt}}}(n)]}$ represents the total number of nodes which are both within the local world and within a radius of $R_{\text{opt}}(n)$ of n, the average

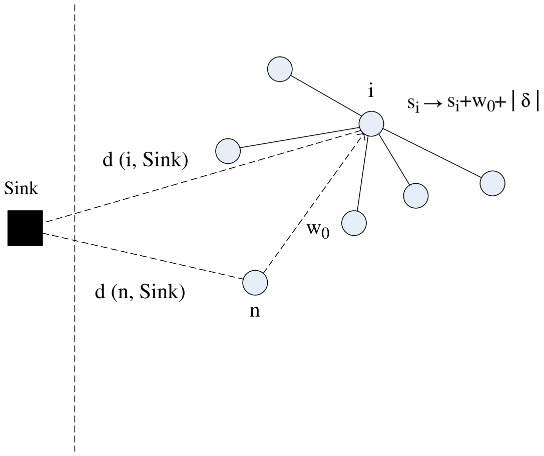

Fig. 4. Change of S_i when the distance from n to the sink node (Sink) is shorter than i.

strength $\langle s_l \rangle$ of these nodes equals to the average strength of all nodes in the network at time t. $\langle w_0 \rangle$ is the average of w_0, then

$$\begin{aligned}\sum_{l\in[\text{local-world}\cap N_{R_{\text{opt}}}(n)]} s_l &= \langle s_l \rangle\, N_{l\in[\text{local-world}\cap N_{R_{\text{opt}}}(n)]} \\ &= \frac{\sum_k s_k(t)}{N_0+t} N_{l\in[\text{local-world}\cap N_{R_{\text{opt}}}(n)]} \\ &\approx \frac{2mt(\delta+\langle w_0\rangle)}{N_0+t} N_{l\in[\text{local-world}\cap N_{R_{\text{opt}}}(n)]}.\end{aligned}$$

The differential equation of the edge weight about time is

$$\begin{aligned}\frac{dw_{ij}(t)}{dt} &= \prod(n\to i)m\delta_i \frac{w_{ij}(t)}{s_i(t)} + \prod(n\to j)m\delta_j \frac{w_{ij}(t)}{s_j(t)} \\ &= m\frac{M}{N_0+t}\cdot\frac{\pi R_{\text{opt}}^2(n)}{WH}\left[\delta_i \frac{s_i(t)}{\sum\limits_{l\in[\text{local-world}\cap N_{R_{\text{opt}}}(n)]} s_l(t)}\cdot\frac{w_{ij}(t)}{s_i(t)} + \delta_j \frac{s_j(t)}{\sum\limits_{l\in[\text{local-world}\cap N_{R_{\text{opt}}}(n)]} s_l(t)}\cdot\frac{w_{ij}(t)}{s_j(t)}\right] \\ &= \frac{2Mm(\delta_i+\delta_j)}{N_0+t}\cdot\frac{\pi R_{\text{opt}}^2(n)}{WH}\cdot\frac{w_{ij}(t)}{\sum\limits_{l\in[\text{local-world}\cap N_{R_{\text{opt}}}(n)]} s_l(t)} \\ &= \frac{2m(\delta_i+\delta_j)w_{ij}(t)}{N_0+t}\cdot\frac{N_0+t}{2mt(\delta+\langle w_0\rangle)}\cdot\frac{\pi R_{\text{opt}}^2(n)}{WH}\cdot\frac{M}{N_{l\in[\text{local-world}\cap N_{R_{\text{opt}}}(n)]}} \\ &= \frac{(\delta_i+\delta_j)w_{ij}(t)}{t(\delta+\langle w_0\rangle)}\cdot\frac{\pi R_{\text{opt}}^2(n)}{WH}\cdot\frac{WH}{\pi R_{\text{opt}}^2(n)} \\ &= \frac{(\delta_i+\delta_j)w_{ij}(t)}{t(\delta+\langle w_0\rangle)},\end{aligned}$$

where δ_i and δ_j are positive or negative increments.

Set

$$\theta = \frac{\delta_i+\delta_j}{\delta+\langle w_0\rangle}. \tag{28}$$

The creation time of e_{ij} is

$$t_{ij} = \max(t_i, t_j), \tag{29}$$

where t_i is the time when i joins the network. According to the definition of edge weight, given the initial condition,

$$w_{ij}(t_{ij}) = \frac{\zeta\,[E_i(t_{ij})E_j(t_{ij})]^{\psi}}{\{[d(i,j)]^2\}^{\eta}[T_{ij}(t_{ij})]^{\xi}} \approx \langle w_0\rangle\,. \tag{30}$$

Therefore, the particular solution of the weight differential equation can be obtained:

$$w_{ij}(t) = \langle w_0\rangle\,(t/t_{ij})^{\theta}. \tag{31}$$

The differential equation of node strength about time is

$$\begin{aligned}
\frac{ds_i}{dt} &= \sum_j \frac{dw_{ij}}{dt} + m\prod(n\to i)\\
&= \frac{(\delta_i+\delta_j)s_i(t)}{(\delta+\langle w_0\rangle)t} + m\frac{M}{N_0+t}\cdot\frac{\pi R^2_{\mathrm{opt}}(n)}{WH}\cdot\frac{s_i(t)}{\sum\limits_{l\in[\text{local-world}\cap N_{R_{\mathrm{opt}}}(n)]} s_l(t)}\\
&= \frac{(\delta_i+\delta_j)s_i(t)}{(\delta+\langle w_0\rangle)t} + m\frac{M}{N_0+t}\cdot\frac{\pi R^2_{\mathrm{opt}}(n)}{WH}\cdot\frac{(N_0+t)s_i(t)}{2mt(\delta+\langle w_0\rangle)N_{l\in[\text{local-world}\cap N_{R_{\mathrm{opt}}}(n)]}}\\
&= \frac{(\delta_i+\delta_j)s_i(t)}{(\delta+\langle w_0\rangle)t} + \frac{s_i(t)}{2(\delta+\langle w_0\rangle)t}\cdot\frac{\pi R^2_{\mathrm{opt}}(n)}{WH}\cdot\frac{WH}{\pi R^2_{\mathrm{opt}}(n)}\\
&= \frac{2(\delta_i+\delta_j)+1}{2\delta+2\,\langle w_0\rangle}\frac{s_i(t)}{t}\\
&= \lambda\frac{s_i(t)}{t},
\end{aligned}$$

where

$$\lambda = \frac{2(\delta_i+\delta_j)+1}{2\delta+2\,\langle w_0\rangle}. \tag{32}$$

The initial condition is

$$s_i(t=i) = \sum_{j=1}^{m} w_{ij} \approx m\,\langle w_0\rangle\,. \tag{33}$$

The particular solution of the node strength differential equation can be obtained:

$$s_i(t) = m\,\langle w_0\rangle\,(t/i)^{\lambda}. \tag{34}$$

The differential equations of node degree about time is

$$\begin{aligned}
\frac{dk_i(t)}{dt} &= m\prod(n\to i)\\
&= \frac{1}{2\delta+2\,\langle w_0\rangle}\cdot\frac{s_i(t)}{t}\\
&= \frac{mt^{\lambda-1}\,\langle w_0\rangle}{i^{\lambda}(2\delta+2\,\langle w_0\rangle)}.
\end{aligned}$$

The initial conditions are

$$k_i(t=i) = m. \tag{35}$$

The particular solution of the node degree differential equation can be obtained:

$$k_i(t) = \frac{m\,\langle w_0\rangle\,(t/i)^{\lambda} - m\,\langle w_0\rangle}{2\delta+1} + m = \frac{s_i(t) - m\,\langle w_0\rangle}{2\delta+1} + m. \tag{36}$$

When calculating the distribution of $k_i(t)$, i is regarded as a random variable which obeys a uniform distribution; that is to say,

$$\rho(i) = \frac{1}{N_0+t}. \tag{37}$$

The derivation of the network degree distribution is as follows:

$$\begin{aligned}
P\{k_i(t) < k\} &= P\left\{\frac{s_i(t) - m\langle w_0\rangle}{2\delta+1} + m < k\right\} \\
&= P\left\{i > t\left[\frac{(2\delta+1)(k-m)}{m\langle w_0\rangle} + 1\right]^{-\frac{1}{\lambda}}\right\} \\
&= 1 - \frac{t}{N_0+t}\cdot\left(\frac{2\delta+1}{m\langle w_0\rangle}\right)^{-\frac{1}{\lambda}}\left[k - \left(m - \frac{m\langle w_0\rangle}{2\delta+1}\right)\right]^{-\frac{1}{\lambda}} \\
P(k,t) &= \frac{\partial P\{k_i(t) < k\}}{\partial k} \\
&= \frac{1}{\lambda}\cdot\frac{t}{(N_0+t)}\cdot\left(\frac{2\delta+1}{m\langle w_0\rangle}\right)^{-\frac{1}{\lambda}}\left[k - \left(m - \frac{m\langle w_0\rangle}{2\delta+1}\right)\right]^{-\left(1+\frac{1}{\lambda}\right)}.
\end{aligned}$$

Let $t \to \infty$. The degree distribution of the WSN achieves a steady state:

$$P(k) = \lim_{t\to\infty} P(k,t) \sim \frac{1}{\lambda}\left(\frac{2\delta+1}{m\langle w_0\rangle}\right)^{-\frac{1}{\lambda}}\left[k - \left(m - \frac{m\langle w_0\rangle}{2\delta+1}\right)\right]^{-\left(1+\frac{1}{\lambda}\right)}. \tag{38}$$

The derivation of the node strength distribution is

$$\begin{aligned}
P\{s_i(t) < s\} &= P\left\{m\langle w_0\rangle (t/i)^{\lambda} < s\right\} \\
&= P\left\{i > t\left(\frac{s}{m\langle w_0\rangle}\right)^{-\frac{1}{\lambda}}\right\} \\
&= 1 - \frac{t}{N_0+t}\left(m\langle w_0\rangle\right)^{\frac{1}{\lambda}} s^{-\frac{1}{\lambda}} \\
P(s,t) &= \frac{\partial P\{s_i(t) < s\}}{\partial s} \\
&= \frac{1}{\lambda}\cdot\frac{t}{N_0+t}\left(m\langle w_0\rangle\right)^{\frac{1}{\lambda}} s^{-\left(1+\frac{1}{\lambda}\right)}.
\end{aligned}$$

Let $t \to \infty$. The strength distribution of the WSN achieves a steady state:

$$P(s) = \lim_{t\to\infty} P(s,t) \sim \frac{1}{\lambda}\left(m\langle w_0\rangle\right)^{\frac{1}{\lambda}} s^{-\left(1+\frac{1}{\lambda}\right)}. \tag{39}$$

The derivation of the edge weight distribution is

$$\begin{aligned}
P\left\{w_{ij}(t) < w\right\} &= P\left\{\langle w_0\rangle\left(\frac{t}{t_{ij}}\right)^{\theta} < w\right\} \\
&= P\left\{t_{ij} > t\langle w_0\rangle^{\frac{1}{\theta}} w^{-\frac{1}{\theta}}\right\} \\
&= 1 - P\left\{t_{ij} \le t\langle w_0\rangle^{\frac{1}{\theta}} w^{-\frac{1}{\theta}}\right\} \\
&= 1 - \frac{t}{N_0+t}\langle w_0\rangle^{\frac{1}{\theta}} w^{-\frac{1}{\theta}} \\
P(w,t) &= \frac{\partial P\left\{w_{ij}(t) < w\right\}}{\partial w} \\
&= \frac{1}{\theta}\cdot\frac{t}{N_0+t}\langle w_0\rangle^{\frac{1}{\theta}} w^{-\left(1+\frac{1}{\theta}\right)}.
\end{aligned}$$

Let $t \to \infty$. The edge weight distribution of the WSN is

$$P(w) = \lim_{t\to\infty} P(w,t) \sim \frac{1}{\theta}\langle w_0\rangle^{\frac{1}{\theta}} w^{-\left(1+\frac{1}{\theta}\right)}. \tag{40}$$

Overall, using analytical methods, we obtain the node strength, degree, and edge weight distributions of the WSN. These distributions all obey a power-law distribution, where

$$\gamma_k = \gamma_s = 1 + \frac{1}{\lambda} = 1 + \frac{2\delta + 2\langle w_0\rangle}{2(\delta_i+\delta_j)+1} \tag{41}$$

$$\gamma_w = 1 + \frac{1}{\theta} = 1 + \frac{\delta + \langle w_0\rangle}{\delta_i+\delta_j}. \tag{42}$$

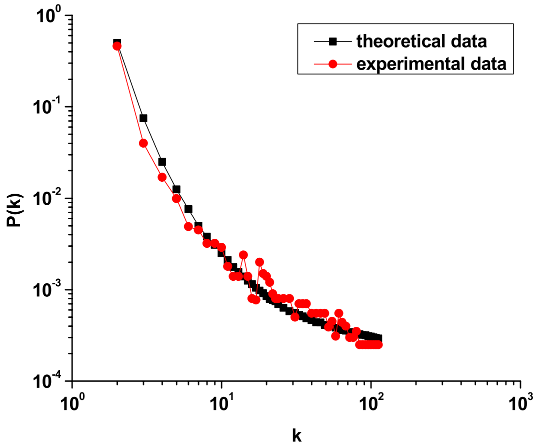

Fig. 5. Comparison of theoretical and experimental $P(k)$.

6. Experimental test and analysis of the results

6.1. Contrast of theoretical and experimental data

Before the experiment, the three functions in (17), (19) and (22) will be quantized. $R_{\text{opt}}(i)$ is calculated as follows [12]:

$$R_{\text{opt}}(i) \sim f_1[d(i, \text{Sink})] = \left[1 - c\frac{d(i, \text{Sink}) - X}{\sqrt{(H/2)^2 + (X+W)^2} - X}\right] d_0, \tag{43}$$

where $c = 0.5$, $d_0 = 87$ m, $X = 50$ m, $W = H = 200$ m.

The edge weight is defined as

$$w_{ij}(t) = \frac{E_i(t)E_j(t)}{[d(i,j)]^2 T_{ij}(t)}, \tag{44}$$

where

$$T_{ij}(t) \sim f_2\left[d(i, \text{Sink}), t\right] = \frac{t}{[d(i, \text{Sink})]^2}. \tag{45}$$

The initial energy of the sensor nodes is $E_i(t=0) = E_j(t=0) = 0.5J$.

$|\delta|$ is set to be directly proportional to the weight:

$$|\delta| \sim f_3[w_{ni}(t)] = \frac{1}{10} w_{ni}(t). \tag{46}$$

Given $N = 400$, $N_0 = 50$, $M = 30$, $m = 3$, and $\langle w_0 \rangle$ is obtained from the final topology using a statistical tool. The distributions of node degree, strength, and edge weight are shown in Figs. 5–7 with coordinates of log 10 scale. Furthermore, the theoretical data are compared with experimental data in the figures.

With $N = 800$ and the sink node moved up vertically by 50 m, the new distributions of the node strength are shown in Fig. 8.

6.2. Analysis of the experimental results

As shown in Figs. 5–7, the experimental distributions of the IOT are consistent with the theoretical distributions in (38)–(40); they follow a power law and show a "tail", which are the basic characteristics of scale-free networks. As shown in Fig. 5, the probability of $P(k) = 1$ is 0.5, indicating that about half of sensors only have one communication link, which forwards data to the next-hop node of the IOT. The probability that a sensor has a large number of neighbors (the degree is larger than 100) is very small, indicating that central nodes are in the minority among all sensors, and the number of communication links connected to one central node is limited. As shown in Fig. 6, the busy communication links with smaller weight in the whole network of the IOT are in the minority, which is the goal of WSN routing strategy design. As shown in Fig. 7, with a higher strength, central nodes have the capacity of forwarding more data. Though common nodes are more numerous than central nodes, their capacity for forwarding data is weak, so the node strength is lower. As shown in Fig. 8, the node strengths are increased by increasing the number of sensors, because the burden of each node decreases, and the capacity for forwarding data in the whole network has been enhanced. When the sink node of the IOT is moved up vertically by 50 m, the distribution of the node strength changes, the probability of higher strengths decreases, and that of the lower ones increases; that is to say, the destruction of symmetry results in the reduction of energy efficiency.

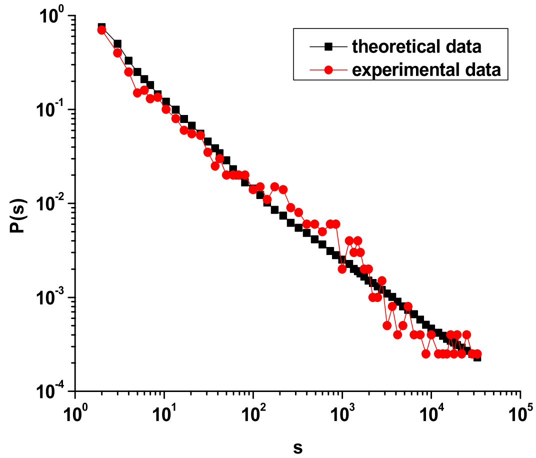

Fig. 6. Comparison of theoretical and experimental $P(s)$.

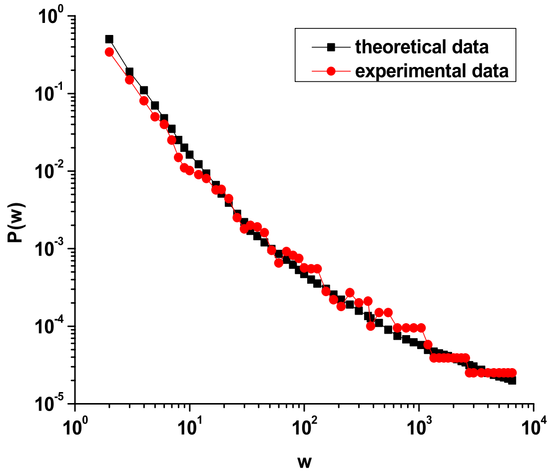

Fig. 7. Comparison of theoretical and experimental $P(w)$.

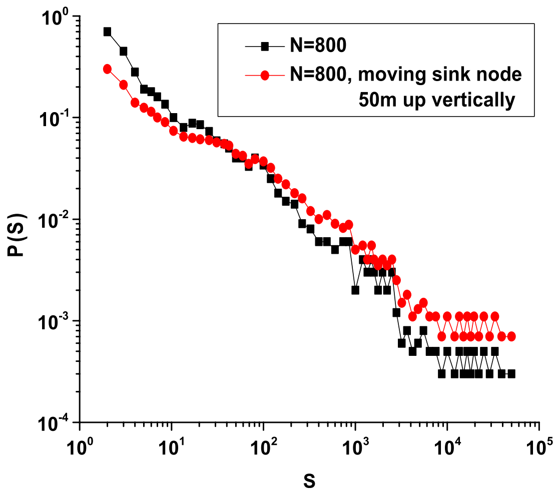

Fig. 8. Comparison of $P(w)$ in different settings.

7. Conclusions

Topological construction of WSNs for application of the IOT is important in designing routing algorithms, security policy, and other further research. In this study, weighted networks and local-world theory are applied in WSNs of the IOT, and the edge weight is newly defined to improve the BBV model. Topology growth and weight dynamics are controlled to get uneven clusters, so that the energy consumption of the entire network of the IOT is balanced, and an energy hole is avoided. The WSN topology evolving by this mechanism possesses properties of scale-free weighted networks of the IOT. Experimental results show that the distributions of node degree, strength, and edge weight follow a power law and represent a "tail", so the topology has robustness and fault tolerance, reducing the probability of successive node breakdown and enhancing the synchronization of the WSNs of the IOT. The definitions of edge weight and node strength are helpful to establish other realistic models of WSNs of the IOT, such as correlation, clustering coefficient, and average shortest path.

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